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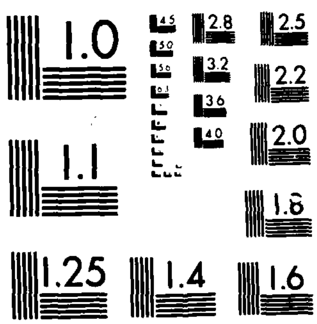

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ELECTROMAGNETIC PENETRATION THROUGH A SLOT INTO A RECTANGULAR WAVEGUIDE

Interim Technical Report No. 3

by

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Electromagnetic penetration into a matched rectangular waveguide through a longitudinal slot in a conducting plane is considered. The problem is divided into two parts, one in half space and another inside the waveguide, by use of the equivalence principle, ~~as described~~ in reference [1]. The resultant equations are the same as those described in reference [2]. The only difference is that the excitation vector is changed from an incident wave inside the waveguide to an incident plane wave in half space. The magnetic current over the slot is obtained from the solution of the matrix equations. The electromag-

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20. ABSTRACT (continued)

magnetic penetration into a matched rectangular waveguide and the electromagnetic reradiation into half space through a slot are computed from the equivalent magnetic current over the slot. The computed results show that a plane wave at normal incidence and a resonated slot near the side of the waveguide broad wall allow orders of magnitude greater power to penetrate into the waveguide than when the slot is not resonated.

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I. INTRODUCTION

The electromagnetic penetration through an aperture in a conducting plane is widely encountered in electromagnetic engineering and is an important problem in the engineering analysis of electromagnetic compatibility and interference. Here we consider the electromagnetic penetration into a matched rectangular waveguide through a longitudinal slot in a conducting plane. At resonance, orders of magnitude greater power can penetrate a slot than the same slot not resonated.

A general formulation for a two-region aperture problem can be derived by use of the equivalence principle [3]. First the operator equations are obtained in terms of an unknown equivalent magnetic current, and then they are reduced to matrix equations via the method of moments [4]. The only coupling is through the slot, whose characteristics are described by two slot admittance matrices, one for half space and another for the inside of the waveguide. These two admittance matrices depend only on the region considered and are independent of the other region. The slot coupling is expressed as the sum of the two independent slot admittance matrices, and the source term is related to the incident magnetic field. The equivalent magnetic current over the slot can be found from the matrix equation.

To obtain the admittance matrix for the inside of the matched waveguide we use Green's functions to represent the fields inside the waveguide, triangle functions for the expansion functions of the equivalent magnetic current, and also for the testing functions.

When the distribution of the equivalent magnetic current is obtained, the other electromagnetic parameters can be derived by known methods [5]. The penetration power from half space into the matched waveguide, or the

reradiation power into half space from the slot, is a measurement which depends quadratically on the equivalent magnetic current over the slot. It can be computed by use of the integral of Poynting vector over the slot. As examples, the computed results for the magnetic current distribution, the resonant length of the slot, the penetration power into the waveguide, the reradiation power gain pattern, and the transmission cross section of the slot are presented.

II. FORMULATION OF THE PROBLEM

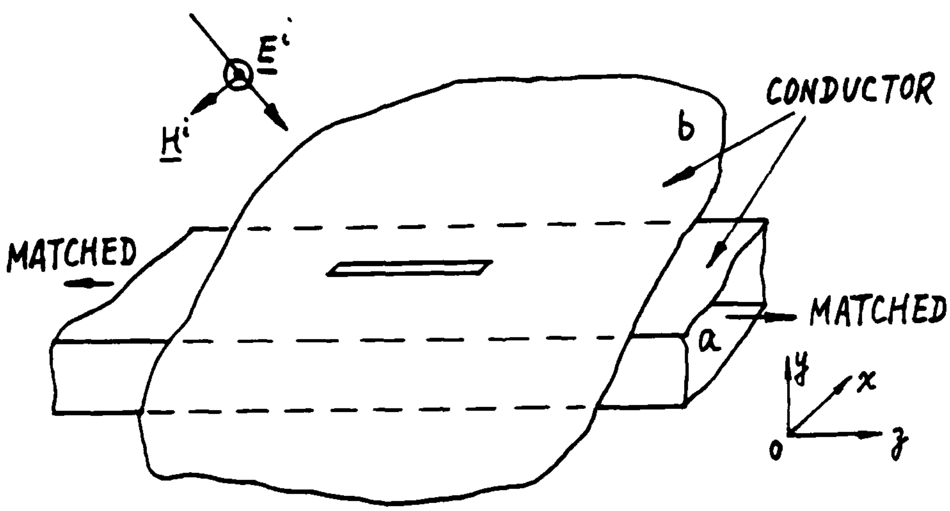
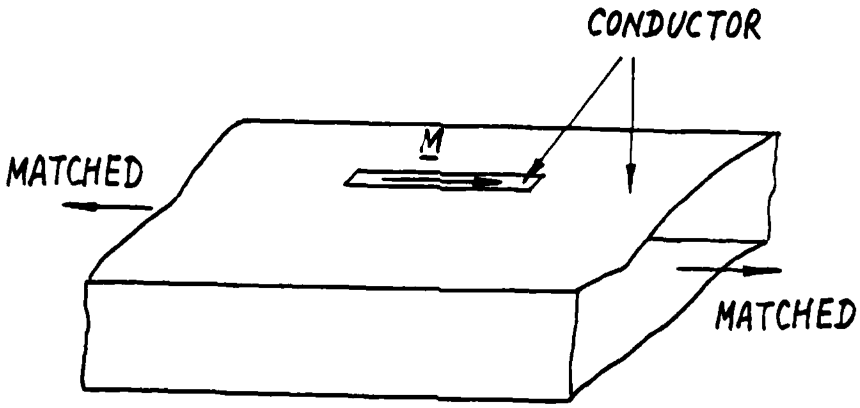


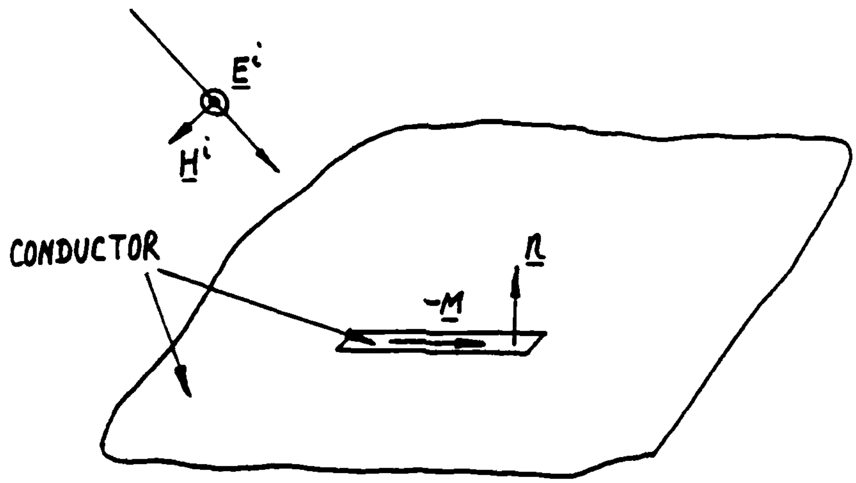
Fig. 1. The original slot coupling problem.

Figure 1 represents the problem of the slot coupling between two regions, one inside the waveguide called region a and another in half space called region b. In region b there is a plane wave as an impressed source, and region a is assumed source free. Each region of Fig. 1 is bounded by an electric conductor. Region a is closed in the x and y directions, and open to infinity in the z direction. Region b is open to infinity.

The problem is primarily that of finding the tangential electric field over the slot and secondarily that of finding the fields throughout space.



(a)



(b)

Fig. 2. The original problem divided into two equivalent problems:

(a) equivalent to region a

(b) equivalent to region b

The equivalence principle, described in reference [1], is used to divide the problem into two equivalent problems as shown in Fig. 2. In region a, the fields are produced by the equivalent magnetic current

$$\underline{M} = \underline{n} \times \underline{E} \tag{1}$$

over the slot region, with the slot covered by an electric conductor. Here $\underline{E}$ is the electric field in the slot of the original problem and $\underline{n}$ is the unit normal pointing outward. In region b, the fields are produced by the incident plane wave and the equivalent magnetic current $-\underline{M}$ over the slot region. The fact that the equivalent magnetic current in region b is the negative of that in region a ensures that the tangential component of the electric field is continuous across the slot. The remaining boundary condition to be applied is continuity of the tangential component of the magnetic field across the slot. This condition gives us an equation from which to calculate the unknown quantity $\underline{M}$.

To express this boundary condition in an equation form, we denote the electric and magnetic fields in region a as

$$\underline{E}^a = \underline{E}^a(\underline{M}) \tag{2}$$

$$\underline{H}^a = \underline{H}^a(\underline{M}) \tag{3}$$

where $\underline{E}^a(\underline{M})$, $\underline{H}^a(\underline{M})$ are the fields produced by $\underline{M}$ in Fig. 2(a), with the slot covered by an electric conductor. Similarly, we denote the electric and magnetic fields in region b as

$$\underline{E}^b = \underline{E}^b(-\underline{M}) + \underline{E}^i = -\underline{E}^b(\underline{M}) + \underline{E}^i \tag{4}$$

$$\underline{H}^b = \underline{H}^b(-\underline{M}) + \underline{H}^i = -\underline{H}^b(\underline{M}) + \underline{H}^i \tag{5}$$

where $\underline{E}^b(-\underline{M})$, $\underline{H}^b(-\underline{M})$ are fields produced by $-\underline{M}$ in Fig. 2(b), and $\underline{E}^i$, $\underline{H}^i$

are fields from the incident plane wave, with the slot covered by an electric conductor. The last equality in (5) is a consequence of the linearity of the $\underline{H}^b$ operator.

To satisfy the boundary condition that the tangential component of $\underline{H}$ must be continuous across the slot, we equate tangential components of (3) and (5), and obtain

$$-\underline{H}_t^a(\underline{M}) - \underline{H}_t^b(\underline{M}) = -\underline{H}_t^i \tag{6}$$

over the slot. The subscript t denotes the tangential component over the slot. This is the basic operator equation for determining the equivalent magnetic current $\underline{M}$.

We next reduce (6) to a matrix equation using the method of moments to obtain an approximate solution. For this, we define a set of expansion functions $\{\underline{M}_n,\ n=1,2,\ldots,NT\}$, and express the magnetic current over the slot as

$$\underline{M} = \sum_n V_n \underline{M}_n \tag{7}$$

where the coefficients V_n are to be determined. Substituting (7) into (6) and using the linearity of the $\underline{H}_t$ operator, we obtain

$$-\sum_n V_n \underline{H}_t^a(\underline{M}_n) - \sum_n V_n \underline{H}_t^b(\underline{M}_n) = -\underline{H}_t^i \tag{8}$$

over the slot. For the slot region, we define a set of testing functions $\{\underline{W}_m,\ m=1,2,\ldots,NT\}$ and a symmetric product

$$\langle A,B\rangle_{slot} = \iint_{slot} \underline{A}\cdot\underline{B}\, ds \tag{9}$$

We take the symmetric product of (8) with each testing function $\underline{W}_m$, and use the linearity of the symmetric product to obtain

$$-\sum_n V_n \langle W_m, H_t^a(\underline{M}_n)\rangle - \sum_n V_n \langle W_m, H_x^b(\underline{M}_n)\rangle = -\langle W_m, H_t^i\rangle$$

$$m = 1,2,\ldots, NT \qquad (10)$$

Solution of this set of linear equations determines the unknown coefficients V_n and the equivalent magnetic current $\underline{M}$ according to (7). Once $\underline{M}$ is known, the fields and field-related parameters may be computed by standard methods.

The above solution can be put into matrix notation as follows. Define a slot admittance matrix for region a as

$$[Y^a] = [-\langle W_m, H_t^a(\underline{M}_n)\rangle]_{N\times N} \qquad (11)$$

and a slot admittance matrix for region b as

$$[Y^b] = [-\langle W_m, H_t^b(\underline{M}_n)\rangle]_{N\times N} \qquad (12)$$

Define a source vector

$$\vec{I}^i = [-\langle W_m, H_t^i\rangle]_{N\times 1} \qquad (13)$$

and a coefficeint vector

$$\vec{V} = [V_n]_{N\times 1} \qquad (14)$$

Equation (10) is equivalent to the matrix equation

$$[Y^a + Y^b]\vec{V} = \vec{I}^i \qquad (15)$$

The resultant voltage vector

$$\vec{V} = [Y^a + Y^b]^{-1}\vec{I}^i \qquad (16)$$

is then the vector of coefficients which gives $\underline{M}$ according to (7).

It is important to note that computation of $[Y^a]$ involves only region a and computation of $[Y^b]$ involves only region b. Hence, we have divided the problem into two parts, each of them may be formulated independently.

III. THE SLOT ADMITTANCE MATRIX INSIDE THE WAVEGUIDE

Figure 3 shows the slot configuration and the coordinate system,

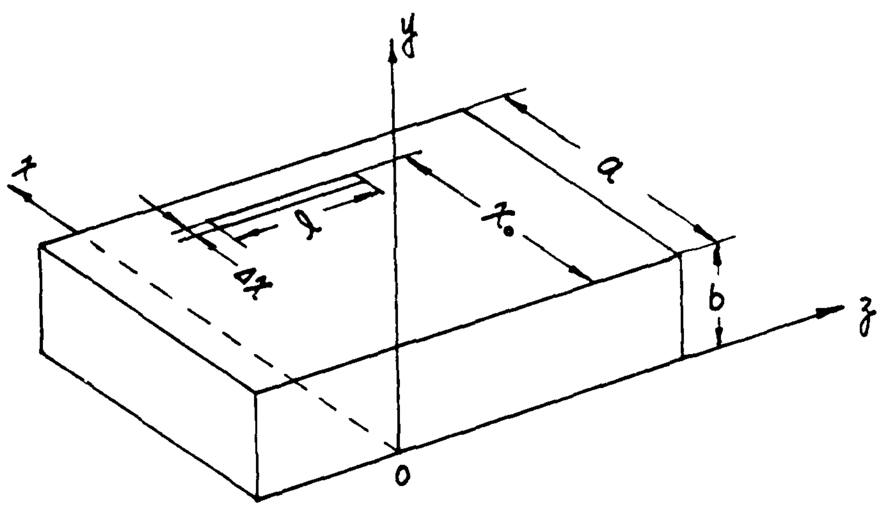


Fig. 3. The slot configuration and the coordinate system.

where a and b represent the dimensions of the cross section of the waveguide. The slot, which is a narrow rectangular aperture oriented along the z direction, is assumed to be in the broad wall of the rectangular waveguide, i.e., in the y=b plane. x_o denotes the distance from the x=0 plane to the middle line of the slot, and is called the displacement of the slot. Δx represents the width of the slot. ℓ

denotes the length of the slot. To apply the method of moments, ℓ is divided into NS subsections. The symbol NT denotes the number of the triangle functions which can be set up on the base of NS subsections.

$$NT = NS - 1 \tag{17}$$

The symbol Δz denotes the length of each subsection,

$$\Delta z = \frac{\ell}{NT+1} \tag{18}$$

We use Galerkin's method to evaluate the slot admittance matrix inside the waveguide, and select triangle functions as expansion and testing functions, i.e.,

$$\underline{W}_p = \underline{M}_p = \underline{U}_z \, T_p(z), \quad p=1,2,\ldots,NT \tag{19}$$

where $T_p(z)$ is a triangle function

$$T_p(z) = \begin{cases} \dfrac{z - (p-1)\Delta z}{\Delta z}, & (p-1)\Delta z < z < p\Delta z \\[2ex] \dfrac{(p+1)\Delta z - z}{\Delta z}, & p\Delta z < z < (p+1)\Delta z \\[2ex] 0 & \text{otherwise} \end{cases} \tag{20}$$

Using the procedure described in reference [2], we evaluate the admittance matrix inside the waveguide as

$$\begin{aligned} Y^a_{ij} &= - \langle W_j, H^a_t(\underline{M}_i) \rangle \\ &= \frac{j\omega\varepsilon}{2ab} \sum_{n=0} A_n(x_o,\Delta x) \sum_{m=0} \frac{\varepsilon_m}{r^3_{mn}(\Delta z)^2} \Big\{ \frac{2}{r^2_{mn}} [3 - 2\, r_{mn}\Delta z + \frac{2}{3} (r_{mn}\Delta z)^3 \\ &\quad - 4\, e^{-r_{mn}\Delta z} + e^{-r_{mn}2\Delta z}] + \frac{1}{k^2} [4(r_{mn}\Delta z + e^{-r_{mn}\Delta z} - 1) \\ &\quad - 2e^{-r_{mn}2\Delta z} (1-e^{r_{mn}\Delta z})^2] \Big\} \end{aligned}$$

$$i = j \tag{21}$$

$$Y_{ij}^{a} = \frac{j\omega\varepsilon}{2ab} \sum_{n=0} A_n(x_o,\Delta x) \sum_{m=0} \frac{\varepsilon_m}{\gamma_{mn}^3(\Delta z)^2} \{\frac{1}{\gamma_{mn}^2}[2(\gamma_{mn}\Delta z-2) + \frac{(\gamma_{mn}\Delta z)^3}{3} + 7e^{-\gamma_{mn}\Delta z}$$

$$- 4e^{-\gamma_{mn}2\Delta z} + e^{-\gamma_{mn}3\Delta z}] - \frac{1}{k^2}[(1-e^{\gamma_{mn}\Delta z})^2(2e^{-\gamma_{mn}2\Delta z} - e^{-\gamma_{mn}3\Delta z}) + 2(1-\gamma_{mn}\Delta z-e^{-\gamma_{mn}\Delta z})]\}$$

$$|i-j| = 1 \qquad (22)$$

$$Y_{ij}^{a} = \frac{j\omega\varepsilon}{2ab} \sum_{n=0} A_n(x_o,\Delta x) \sum_{m=0} \frac{\varepsilon_m}{\gamma_{mn}^3(\Delta z)^2} \{\frac{e^{-\gamma_{mn}(p-1)\Delta z}}{\gamma_{mn}^2}[6+e^{\gamma_{mn}2\Delta z} - 4e^{\gamma_{mn}\Delta z}$$

$$-4e^{-\gamma_{mn}\Delta z} + e^{-\gamma_{mn}2\Delta z}]$$

$$- \frac{e^{-\gamma_{mn}p\Delta z}}{k^2}(1 - e^{\gamma_{mn}\Delta z})^2 (2-e^{-\gamma_{mn}\Delta z} - e^{\gamma_{mn}\Delta z})\}$$

$$|i-j| \geq 1,\ p \geq 3 \qquad (23)$$

where

$$A_n(x_o,\ x) = \begin{cases} (\Delta x)^2 & ,\quad n = 0 \\ 8(\frac{a}{n\pi})^2 [\cos(\frac{n\pi}{a} x_o) \sin(\frac{n\pi}{2a}\Delta x)]^2 & ,\quad n > 0 \end{cases} \qquad (24)$$

$$k^2 = \omega^2\varepsilon\mu = (\frac{2\pi}{\lambda_o})^2\varepsilon_r \qquad (25)$$

$$\gamma = \gamma_{mn} = \begin{cases} j\sqrt{k^2 - k_c^2} & ,\ k^2 > k_c^2 \\ \sqrt{k_c^2 - k^2} & ,\ k^2 < k_c^2 \end{cases} \qquad (26)$$

ε_r is the relative permittivity and ε_m is Neuman's number. a and b are the lengths of the broad and narrow sides of the waveguide cross section, respectively.

IV. THE SLOT ADMITTANCE MATRIX FOR HALF SPACE

A good solution to the slot admittance matrix for half space, with the formulation and the computer program, is available. The details can be found in reference [6].

V. THE EXCITATION VECTOR FOR A PLANE WAVE

Suppose the excitation is a plane wave incident on the slot from half space, as shown in Fig. 4. The slot is along the z direction, y is

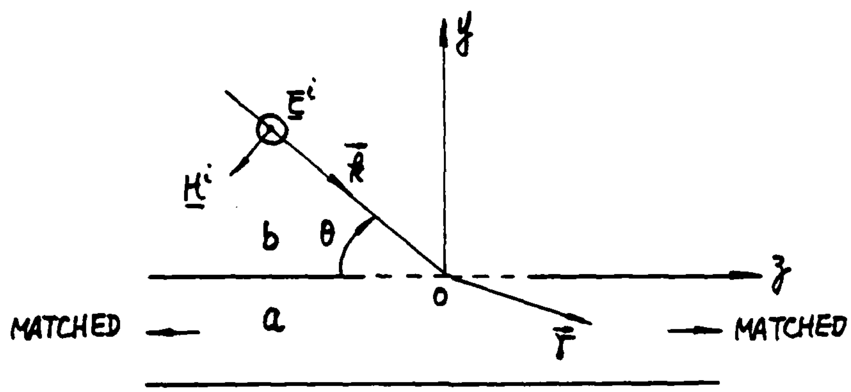


Fig. 4. Plane wave incidence.

normal to the broad wall of the waveguide, which is matched at both ends. The origin of the coordinate system is at the center of the slot. θ is an angle from -z direction to incidence direction. $\underline{H}$ is the magnetic field of the plane wave and is in the y-z plane. $\underline{E}$ is the electric field of the plane wave and is perpendicular to the y-z plane. $\vec{k}$ represents the propagation vector of magnitude $2\pi/\lambda$ and pointing in the direction of propagation. $\vec{r}$ is the radius vector to an arbitrary field point. Let

$$\vec{r} = \underline{U}_y y + \underline{U}_z z \tag{28}$$

$$\vec{k} = \underline{U}_y k_y + \underline{U}_z k_z \tag{29}$$

where

$$k_y = - k \sin \theta \tag{30}$$

$$k_z = \ k \cos \theta$$

The equiphase plane equation is

$$e^{-j\vec{k}\cdot\vec{r}} = e^{-jk_y y} e^{-jk_z z} \tag{32}$$

A unit plane wave is given by

$$\underline{E} = \underline{U}_x e^{-jk(-y \sin\theta + z \cos \theta)}$$

$$\underline{H} = \frac{1}{\eta} (\underline{U}_y \cos\theta + \underline{U}_z \sin \theta) e^{-jk(-y \sin\theta + z \cos \theta)} \tag{33}$$

where η is the intrinsic impedance of the medium in half space.

The impressed field $\underline{H}_t^i$ used in (13) is the tangential magnetic field due to the incident plane wave with the slot covered by an electric conductor. In this case the y=0 plane is a complete conducting plane and image theory can be applied. The tangential component of the magnetic field over the y=0 plane when the slot is covered by a conductor is just twice what it is due to same plane wave in free space, or

$$\underline{H}_t^i = 2\underline{H}_t^{io} \tag{34}$$

where $\underline{H}_t^{io}$ is the tangential component of the magnetic field over the slot due to the plane wave in free space. The components of the excitation vector $\vec{I}^i$ defined by (13) are

$$I_m^i = -2 \iint\limits_{slot} \underline{W}_m \cdot \underline{H}_t^{io} \, ds$$

where $\underline{W}_m$ is the mth testing function. If a Galerkin solution is used, that is, if $\underline{W}_n = \underline{M}_n = \underline{U}_z T_n(z)$, it then follows

$$I_m^i = - \langle W_m, H_t^i \rangle$$

$$= - \iint\limits_{slot} 2 \, T_m(z) \, \underline{U}_z \cdot \underline{U}_y \frac{\sin \theta}{\eta} e^{-jkz \cos \theta} ds$$

$$= - \frac{2 \sin \theta}{\eta} \int_{x_o - \Delta x/2}^{x_o + \Delta x/2} dx \int_{(m-1)\Delta z}^{(m+1)\Delta z} T_m(z) \, e^{-jkz \cos \theta} dz$$

$$= - \frac{2\Delta x \sin \theta}{\eta} \int_{(m-1)\Delta z}^{(m+1)\Delta z} T_m(z) \, e^{-jkz \cos \theta} dz$$

$$\int_{(m-1)\Delta z}^{(m+1)\Delta z} T_m(z) \, e^{-jkz \cos \theta} dz = \frac{e^{-jkm\Delta z \cos \theta}}{(k \cos \theta)^2 \Delta z} (2 - e^{jk\Delta z \cos \theta} - e^{-jk\Delta z \cos \theta}$$

$$I_m^i = - \frac{2\Delta x \sin \theta}{\eta k^2 \cos^2 \theta \Delta z} (2 - e^{jk\Delta z \cos \theta} - e^{-jk\Delta z \cos \theta}) \, e^{-jkm\Delta z \cos \theta}$$

$$m=1,2,\ldots, NT \qquad (35)$$

When $\theta = 0$, we have

$$I_m^i = 0 \quad , \quad m=1,2,\ldots, NT \qquad (36)$$

When $\theta = \frac{\pi}{2}$, we have

$$I_m^i = - \frac{2\Delta x \Delta z}{\eta} , \quad m=1,2,\ldots, NY \qquad (37)$$

VI. THE EQUIVALENT MAGNETIC CURRENT OVER THE SLOT

We can compute the coefficient vector $\vec{V}$ from Y^a_{ij}, Y^b_{ij} and $\vec{I}^i$, where Y^a_{ij} is the admittance matrix inside the waveguide described by (21), (22), and (23), Y^b_{ij} is the admittance matrix for half space described by reference [6], and $\vec{I}^i$ is the excitation vector from the plane wave described by (35), (36), and (37). For $\theta = \pi/6$, $\pi/4$, $\pi/3$, $\pi/2$, $2\pi/3$, $3\pi/4$, and $5\pi/6$, the computed results $|V_k|$ are listed in Table 1 and drawn in Fig. 5 and Fig. 6. The other parameters are $a = .02286$, $b = .01016$, $x_o = .01448$, $w = .00076$, $\lambda_o = .032$, $m = n = 20$, $NT = 9$. The length of the slot is adjusted to all V_k have approximately the same phase.

From Fig. 5 and Fig. 6, it is seen that the magnitude of the equivalent magnetic current over the slot has a maximum value when the angle of incidence plane wave is equal to $\pi/2$. The distribution of the magnitude of the equivalent magnetic vurrent over the slot is symmetric with respect to the center of the slot when $\theta = \pi/2$. The distribution curves of the magnitude of the equivalent magnetic current over the slot for $\theta = \pi/3$ and $2\pi/3$, $\pi/4$ and $3\pi/4$, $\pi/6$ and $5\pi/6$ are also symmetric with respect to the center of the slot.

Suppose $NT = 9$, $m = n = 20$, $a = .02286$, $b = .01016$, $\lambda_o = .032$, $\varepsilon_r = 1.0$, $w = .00076$, $x_o = .01448$, and $\theta = \pi/2$. Here NT is the number of the expansion and testing function, m and n are the numbers of the waveguide modes taken into account, a is the length of the broad side of the waveguide cross section, b is the length of the narrow side, λ_o is the wavelength in free space, ε_r is the relative permittivity of the medium, w is the width of the slot, x_o is the displacement of the slot, and θ is the incident angle of the plane wave. We compute

Table 1. The computer results of $|V_k|$

$|V_k|$ \ θ	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	$2\pi/3$	$3\pi/4$	$5\pi/6$
$|V_1|$	$.9382\times10$	$.1432\times10^2$	$.1898\times10^2$	$.2429\times10^2$	$.2073\times10^2$	$.1625\times10^2$	$.1097\times10^2$
$|V_2|$	$.1598\times10^2$	$.2430\times10^2$	$.3203\times10^2$	$.4046\times10^2$	$.3413\times10^2$	$.2663\times10^2$	$.1793\times10^2$
$|V_3|$	$.2127\times10^2$	$.3221\times10^2$	$.4224\times10^2$	$.5276\times10^2$	$.4403\times10^2$	$.3421\times10^2$	$.2295\times10^2$
$|V_4|$	$.2485\times10^2$	$.3747\times10^2$	$.4890\times10^2$	$.6043\times10^2$	$.4991\times10^2$	$.3861\times10^2$	$.2581\times10^2$
$|V_5|$	$.2642\times10^2$	$.3969\times10^2$	$.5154\times10^2$	$.6304\times10^2$	$.5154\times10^2$	$.3968\times10^2$	$.2642\times10^2$
$|V_6|$	$.2581\times10^2$	$.3861\times10^2$	$.4991\times10^2$	$.6043\times10^2$	$.4890\times10^2$	$.3747\times10^2$	$.2485\times10^2$
$|V_7|$	$.2295\times10^2$	$.3421\times10^2$	$.4403\times10^2$	$.5276\times10^2$	$.4224\times10^2$	$.3221\times10^2$	$.2127\times10^2$
$|V_8|$	$.1793\times10^2$	$.2663\times10^2$	$.3413\times10^2$	$.4046\times10^2$	$.3203\times10^2$	$.2430\times10^2$	$.1527\times10^2$
$|V_9|$	$.1097\times10^2$	$.1625\times10^2$	$.2073\times10^2$	$.2429\times10^2$	$.1898\times10^2$	$.1432\times10^2$	$.9382\times10$

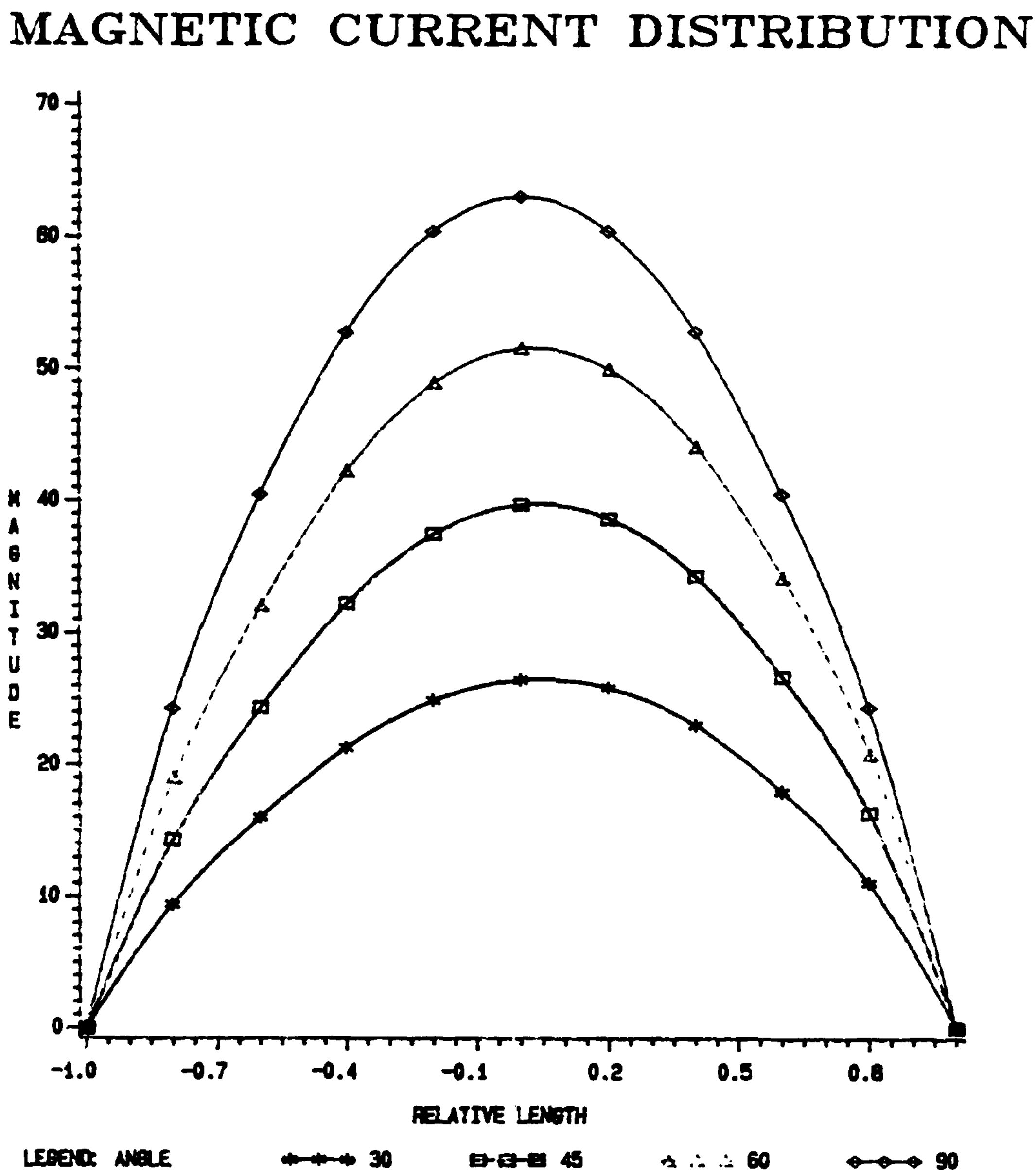


FIG.5 COEFFICIENT VECTOR VERSUS LENGTH

MAGNETIC CURRENT DISTRIBUTION

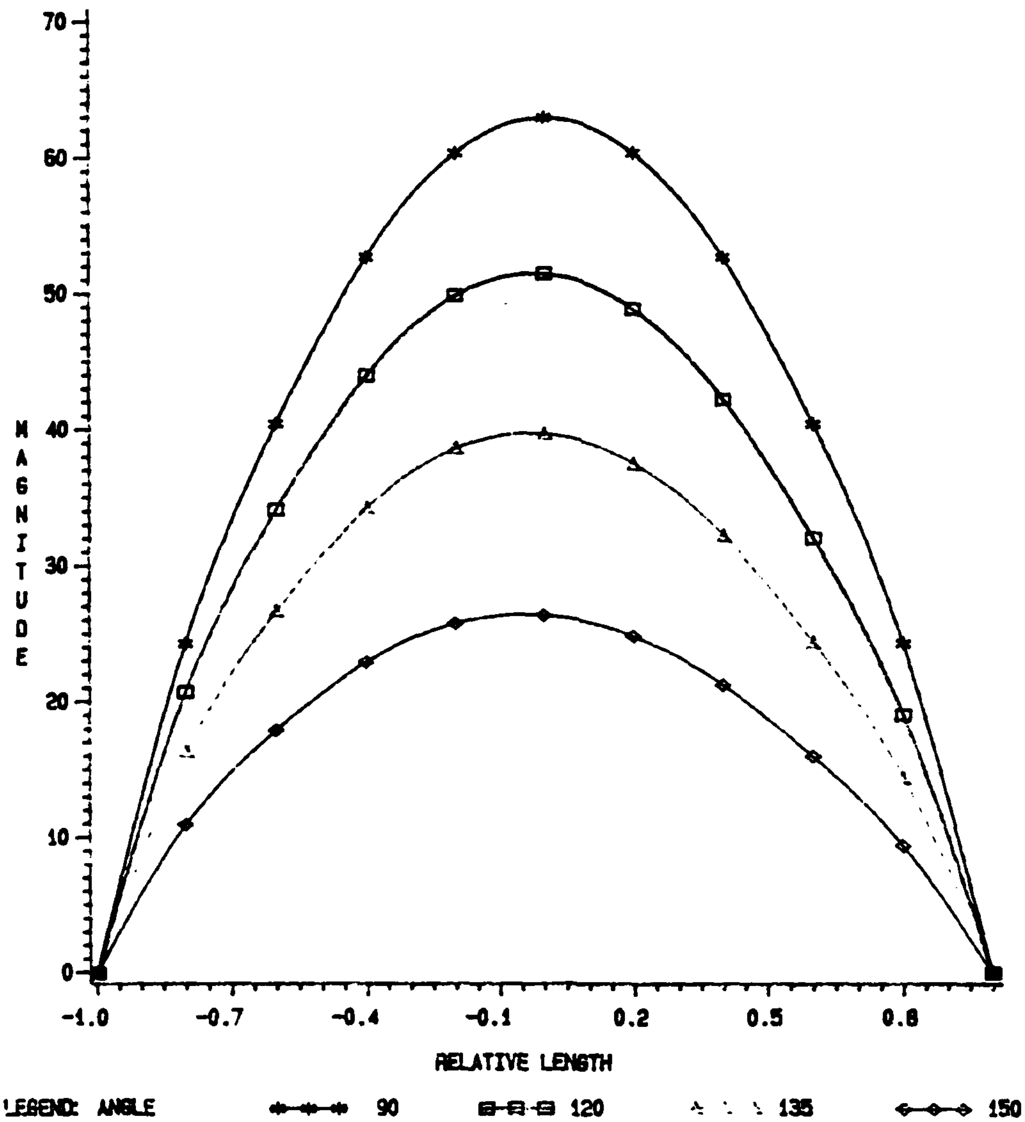


FIG.6 COEFFICIENT VECTOR VERSUS LENGTH

the coefficient vectors of the magnetic currents for various slot lengths, keeping other parameters unchanged. From the computed results we find that when the length of the slot reaches a special value, the magnitudes of the components of the coefficient vector obtain their maximum values and the phases of the components of the coefficient vector are almost the same. We call this length of the slot "resonant length," and denote it by L_{res}. For the present case, it is equal to 0.47584. Now we take $RAT_{1,2,3} = .95\ L_{res}$, L_{res}, $1.05\ L_{res}$, where RAT represents the ratio of the length of the slot to the wavelength in free space, compute the coefficient vectors of the magnetic currents and obtain the results listed in Table 2 and drawn in Fig. 7.

Table 2. The computed results of V_k for $RAT_{1,2,3} = (.95, 1, 1.05)\ L_{res}$

RAT / k	$0.95\ L_{res}$		L_{res}		$1.05\ L_{res}$	
	$\lvert V_k \rvert$	ϕ_k	$\lvert V_k \rvert$	ϕ_k	$\lvert V_k \rvert$	ϕ_k
1	$.2075\times10^2$	36.40	$.2429\times10^2$	.2262	$.1974\times10^2$	-30.56
2	$.3419\times10^2$	36.29	$.4046\times10^2$	.0786	$.3324\times10^2$	-30.74
3	$.4432\times10^2$	36.21	$.5276\times10^2$	-.0145	$.4362\times10^2$	-30.86
4	$.5059\times10^2$	36.17	$.6042\times10^2$	-.0676	$.5014\times10^2$	-30.92
5	$.5272\times10^2$	36.16	$.6304\times10^2$	-.0848	$.5236\times10^2$	-30.95
6	$.5059\times10^2$	36.17	$.6042\times10^2$	-.0676	$.5014\times10^2$	-30.92
7	$.4432\times10^2$	36.21	$.5276\times10^2$	-.0145	$.4362\times10^2$	-30.86
8	$.3419\times10^2$	36.29	$.4046\times10^2$	.0786	$.3324\times10^2$	-30.74
9	$.2075\times10^2$	36.40	$.2429\times10^2$	.2262	$.1974\times10^2$	-30.56

MAGNETIC CURRENT DISTRIBUTION

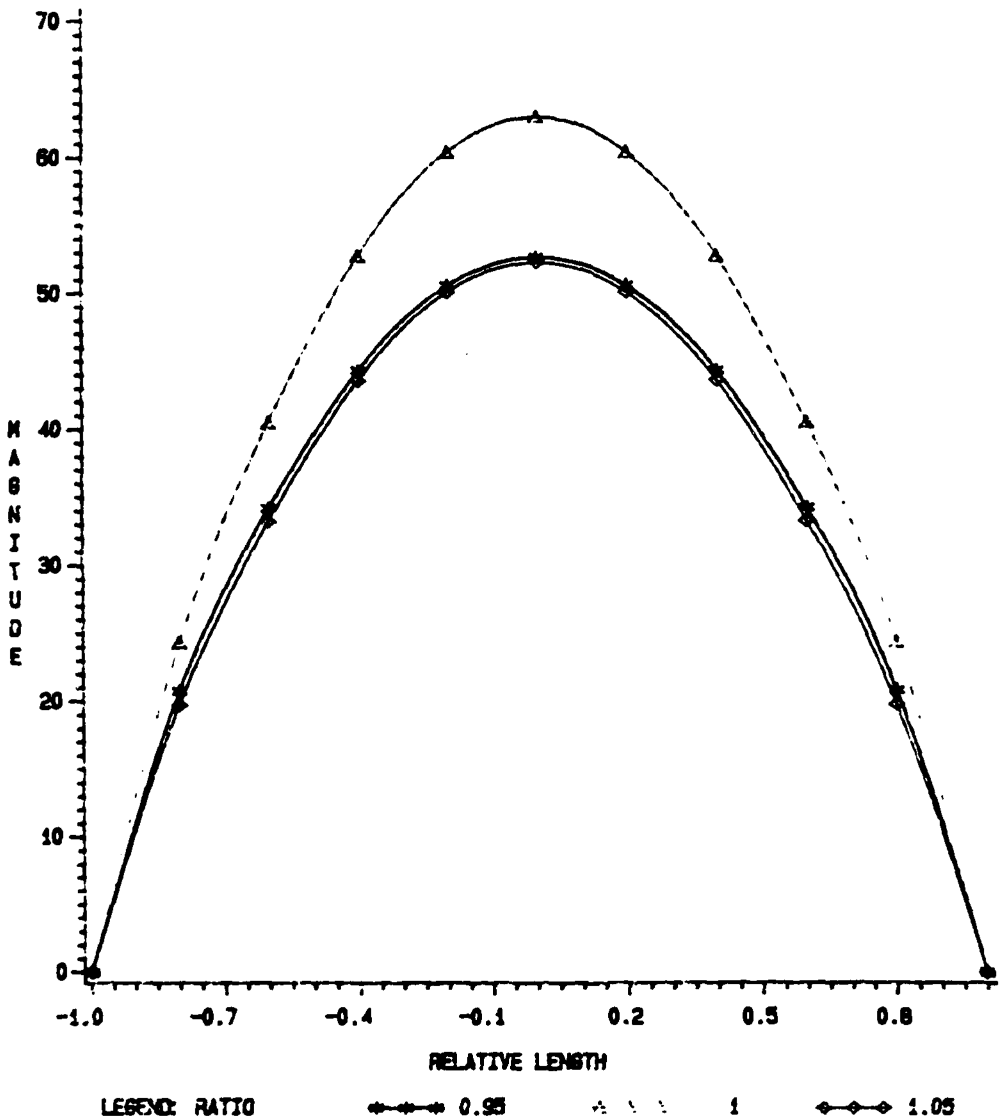


FIG.7 COEFFICIENT VECTOR VERSUS LENGTH

When the incident angles of the plane wave have different values and the other parameters are kept unchanged, we obtain the different resonant lengths of the slot. The computed results of resonant lengths for $\theta = \pi/6$, $\pi/4$, $\pi/3$, $\pi/2$, $2\pi/3$, $3\pi/4$, and $5\pi/6$ are listed in Table 3.

Table 3. The resonant lengths for different incident angles

θ	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	$2\pi/3$	$3\pi/4$	$5\pi/6$
L_{res}	.47680	.47644	.47612	.47584	.47612	.47644	.47680

From Table 3, we see that the resonance lengths are little changed for different incidence angles. The resonant lengths are the same for $\theta = \pi/6$ and $5\pi/6$, $\pi/4$ and $3\pi/4$, and $\pi/3$ and $2\pi/3$.

VII. POWER PENETRATION INTO THE MATCHED WAVEGUIDE

The power transmitted through the slot into the waveguide is a measurement which depends quadratically on the sources. The complex power P_c through the slot is given by

$$P_c = - \iint_{slot} \underline{E} \times \underline{H}^* \cdot \underline{n} \, ds \tag{38}$$

where $\underline{H}^*$ is the conjugate of the magnetic field in the slot and $\underline{n}$ denotes the unit normal to the broad wall of the waveguide. Using the vector identity $\underline{B} \times \underline{C} \cdot \underline{A} = \underline{A} \times \underline{B} \cdot \underline{C}$, we have

$$P_c = - \iint_{slot} \underline{n} \times \underline{E} \cdot \underline{H}^* \, ds \tag{39}$$

Substituting (1) into (39), we have

$$P_c = -\iint_{slot} \underline{M} \cdot \underline{H}^* \, ds \tag{40}$$

This involves only the tangential components of $\underline{H}$, which in region a we denote by $\underline{H}_t^a(\underline{M})$. For $\underline{M}$ we use (7) and obtain

$$\underline{H}_t^a(\underline{M}) = \sum_{n=1}^{NT} V_n \underline{H}_t^a(\underline{M}_n) \tag{41}$$

Substituting (41) and (7) into (40), we obtain

$$P_c = -\sum_{m=1}^{NT} \sum_{n=1}^{NT} V_m V_n^* \iint_{slot} \underline{M}_n \cdot \underline{H}^{a*}(\underline{M}_n) \, ds \tag{42}$$

From the definition of the admittance matrix by (11), we have

$$Y_{mn}^a = -\iint_{slot} \underline{M}_m \cdot \underline{H}^a(\underline{M}_n) \, ds \tag{43}$$

These are the elements of the admittance matrix of the slot inside the waveguide. Since $\underline{M}_m$ is a triangle function which is real, we have

$$Y_{mn}^{a*} = -\iint_{slot} \underline{M}_m \cdot \underline{H}^{a*}(\underline{M}_n) \, ds \tag{44}$$

Substituting (44) into (42), we obtain

$$P_c = \sum_{m=1}^{NT} \sum_{n=1}^{NT} V_m V_n^* Y_{mn}^{a*} \tag{45}$$

This can be written in matrix form as

$$P_c = \tilde{\vec{V}} \, [Y^a]^* \vec{V}^* \tag{46}$$

Here $\vec{V}$ is defined by (14) and the tilde denotes transpose. The time average power transmitted into the waveguide through the slot is given by

$$P_{ct} = \mathrm{Re}(P_c) \tag{47}$$

From (47) and (46), the power transmitted into the waveguide depends on the magnetic current distribution over the slot and the generalized slot admittance matrix inside the waveguide. Table 4 and Fig. 9 show the powers transmitted into the waveguide through the slot for various slot lengths under condition 1, where NT=9, m=n=20, a = .02286, b = .01016, ε_r = 1.0, λ_o = .032, w = .00076, x_o = .01448, and $\theta = \pi/2$. It can be seen that the power P_{ct} reaches its maximum when the length of the slot is equal to .4819λ_o, which we can call resonant length 2 of the slot inside the waveguide.

Table 4. The power penetration versus slot length

ℓ/λ_o	.09638	.1928	.2891	.3855	.4819
P_{ct}	$.2104\times10^{-11}$	$.8172\times10^{-10}$	$.1107\times10^{-8}$	$.1519\times10^{-7}$	$.2345\times10^{-6}$
ℓ/λ_o	.5783	.6746	.7710	.8674	.9638
P_{ct}	$.5048\times10^{-7}$	$.2669\times10^{-7}$	$.1995\times10^{-7}$	$.1709\times10^{-7}$	$.1581\times10^{-7}$

Figure 8 shows the powers transmitted into the waveguide through the slot for various slot lengths under condition 2, where x_o = .0115, x_o = .0125, x_o = .0145, x_o = .0185, x_o = .0225, and the other parameters are the same as under condition 1.

Figure 9 shows the powers transmitted into the waveguide through the slot for various slot lengths under condition 3, where $\theta = \pi/6, \pi/4, \pi/3, \pi/2$, and the other parameters are the same as under condition 1.

From Fig. 8 and Fig. 9, we see that when $\theta = \pi/2$ and x_o = .02250, i.e., when the incident plane wave is perpendicular to the slot, the slot is placed at the side of the waveguide broad wall, and the slot length equals resonant length 2, the power penetration into the waveguide through the slot

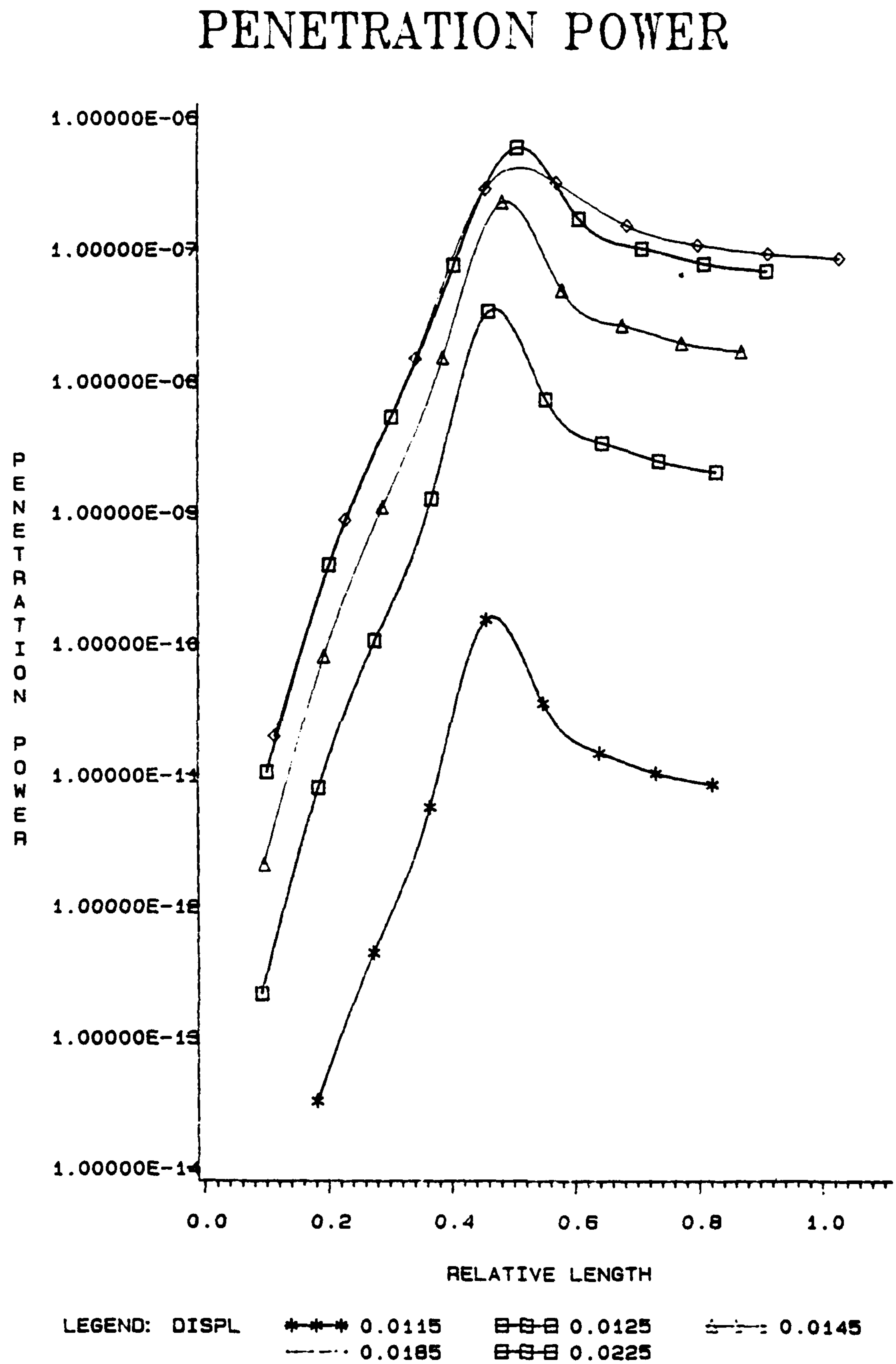


FIG.8. PENETRATION POWER VERSUS LENGTH

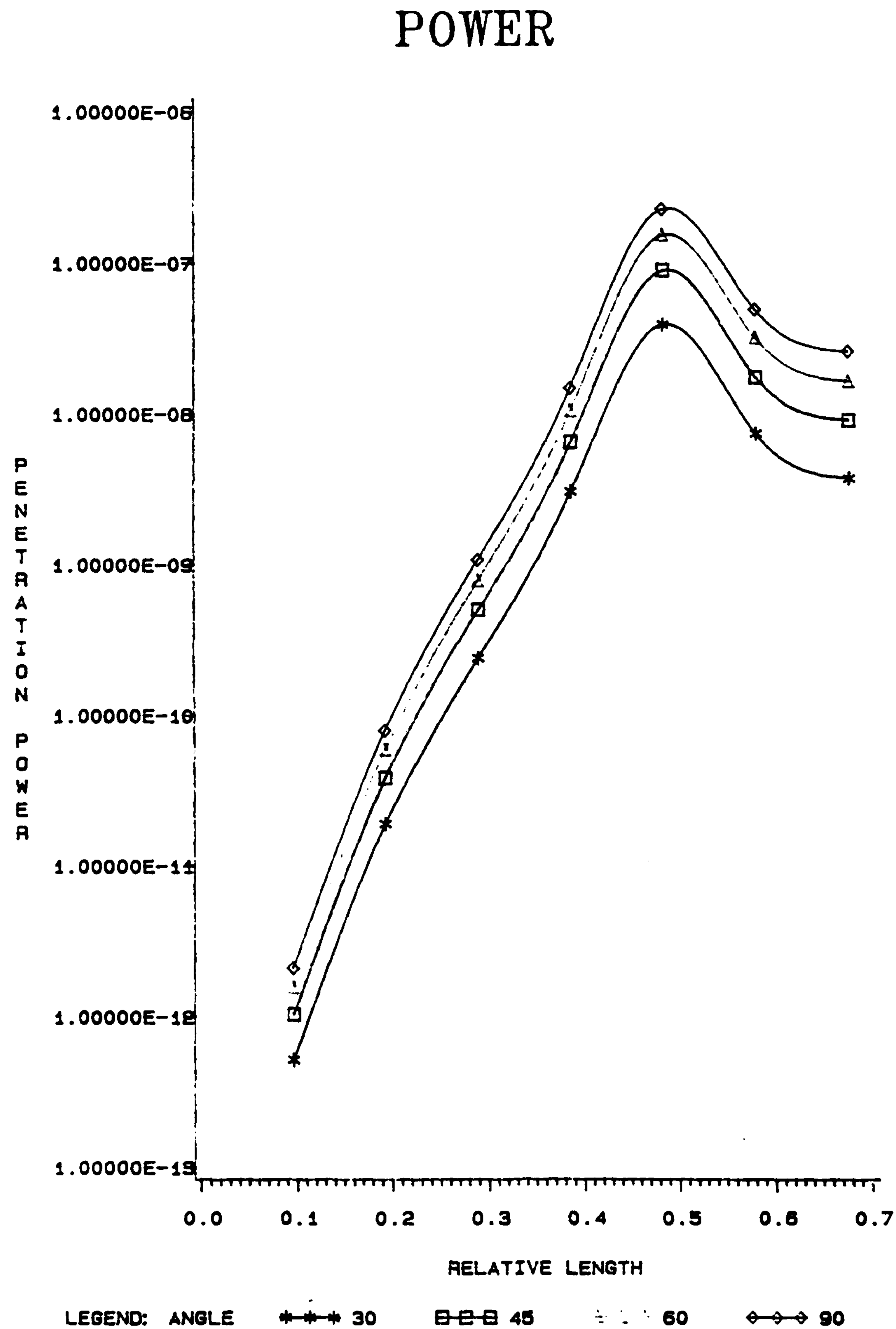


FIG.8. PENETRATION POWER VERSUS LENGTH

reaches a maximum. When the slot length is longer than the resonant one, the power penetration through the slot placed somewhere between the side and the middle line of the broad wall reaches a maximum.

Similarly, P_h denotes the power reradiated into half space from the slot. The complex power P_h given by

$$P_h = \iint_{slot} \underline{E} \cdot \underline{H}^* \cdot \underline{n}\, ds$$

$$= \iint_{slot} \underline{M} \cdot \underline{H}^*\, ds \tag{48}$$

Here $\underline{H}$ is produced by $-\underline{M}$ in half space, denoted by

$$\underline{H} = \underline{H}^b(-\underline{M}) = -\underline{H}^b(\underline{M}) \tag{49}$$

Substituting (49) and (7) into (48), we obtain

$$P_h = -\sum_{m=1}^{NT} \sum_{n=1}^{NT} V_m V_n^* \iint_{slot} \underline{M}_m \cdot \underline{H}^{b*}(\underline{M}_n)\, ds \tag{50}$$

From the definition of the admittance matrix by (12), we have

$$Y_{mn}^b = -\iint_{slot} \underline{M}_m \cdot \underline{H}^b(\underline{M}_n)\, ds \tag{51}$$

Since $\underline{M}_m$ is a real quantity, we have

$$Y_{mn}^{b*} = -\iint_{slot} \underline{M}_m \cdot \underline{H}^{b*}(\underline{M}_n)\, ds \tag{52}$$

Substituting (52) into (50), we obtain

$$P_h = \sum_{m=1}^{NT} \sum_{n=1}^{NT} V_m V_n^* Y_{mn}^{b*} \tag{53}$$

This can be written in matrix form as

$$P_h = \tilde{\vec{V}}[Y^b]*V* \tag{54}$$

The time average power reradiated into half space from the slot is given by

$$P_{ht} = \mathrm{Re}(P_h) \tag{55}$$

Figure 10 shows the powers reradiated into half space from the slot for various slot lengths under condition 1. The power P_{ht} reaches its maximum when the slot length is equal to $.47188\lambda_o$, which we can call resonant length 1 of the slot in half space.

We have defined three resonant lengths of the slot: L_{res1} denotes resonant length 1 of the slot and represents that for which the reradiated power into half space P_{ht} from the slot reaches its maximum. L_{res2} denotes resonant length 2 of the slot and represents that for which the power penetrating into the waveguide P_{ct} through the slot reaches its maximum. L_{res} denotes the resonant length of the slot and represents that the total power, $P_o = P_{ct} + P_{ht}$, reaches its maximum. These three resonant lengths of the slot are different but are close to each other. When the length of the slot equals the resonant length, i.e. $L = L_{res}$, the total power produced by $\underline{M}$ reaches the maximum and the imaginary part of the power is equal to zero. Also, the amplitudes of the components of the coefficient vector reach their maximum, and the phases of the components of the coefficient vector are close to zero (when $\theta = \pi/2$).

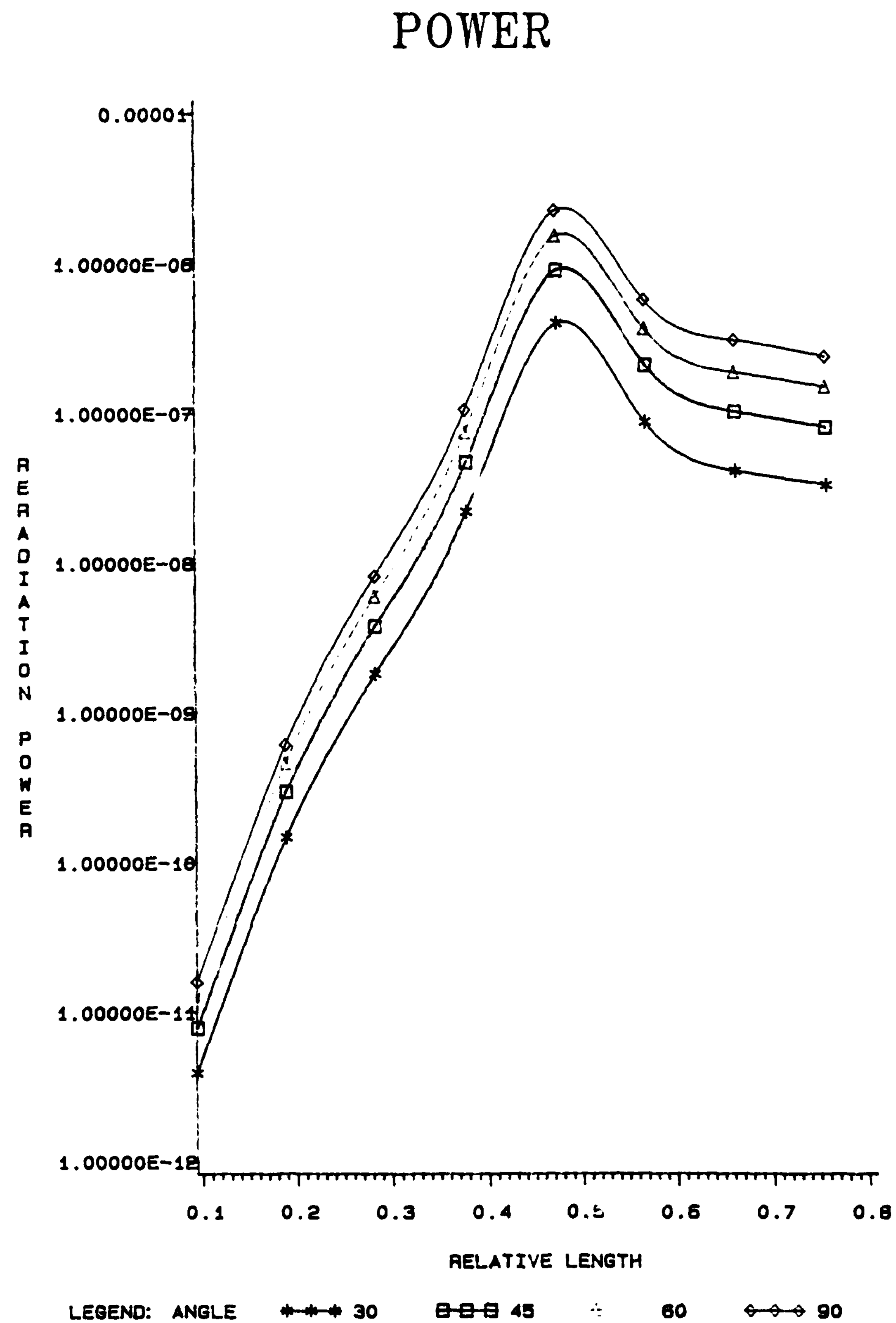


FIG.10 RERADIATION POWER VERSUS LENGTH

VIII. THE POWER GAIN PATTERN

The power gain pattern G of the slot is defined as the ratio of the radiation intensity in a given direction to the radiation intensity which would exist if the power $Re(P_h)$ were radiated uniformly over half space, or

$$G = \frac{2\pi r_m^2 \, \eta |H_m|^2}{Re(P_h)} \tag{56}$$

Here $Re(P_h)$ is given by (54), r_m is the distance to an arbitrary field point in half space, and H_m is a component of the magnetic field on the radiation sphere produced by the slot. This component of the magnetic field can be obtained by placing a magnetic dipole $K\underline{\ell}_m$ at $\underline{r}_m$ and applying the reciprocity theorem to its field and to the original field [1, Section 3-8]. This situation is shown in Fig. 11. Applying the reciprocity theorem, we have

$$H_m K\ell_m = - \iint_{slot} \underline{M} \cdot \underline{H}^m \, ds \tag{57}$$

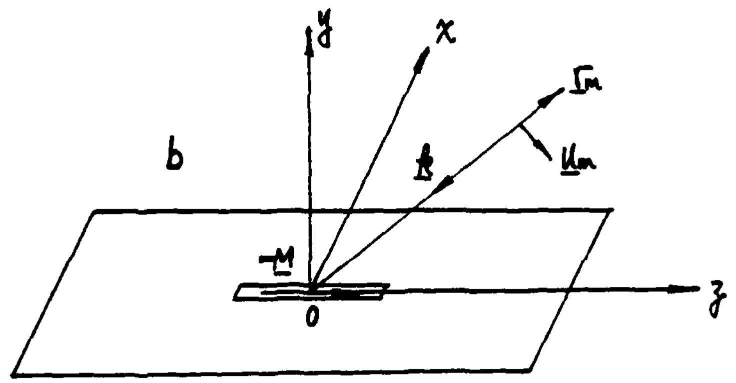


Fig. 11. The measurement of the far-zone field.

where $\underline{H}^m$ is the magnetic field from $K\underline{\ell}_m$ in the presence of a complete conducting plane, and H_m is the component of the magnetic field in the direction of $K\underline{\ell}_m$ at $\underline{r}_m$ due to $-\underline{M}$ in the presence of the conducting plane. Substituting (7) into (57), we have

$$H_m K\ell_m = \sum_n V_n \iint_{slot} - \underline{M}_n \cdot \underline{H}^m \, ds \tag{58}$$

This can be written in matrix form as

$$H_m K\ell_m = \tilde{\vec{I}}^m \vec{V} \tag{59}$$

where

$$I_n^m = - \iint_{slot} \underline{M}_n \cdot \underline{H}^m \, ds \tag{60}$$

Substituting (19) into (60), we obtain

$$I_n^m = - \int_{x_o-\Delta x/2}^{x_o+\Delta x/2} dx \int_{(n-1)\Delta z}^{(n+1)\Delta z} T_n(z)\, H_z^m \, dz \tag{61}$$

where H_z^m denotes the z-direction component of the magnetic field on the slot from $K\underline{\ell}_m$. Since the x-z plane is a complete conducting one, we obtain

$$H_z^m = 2H_z^{mo} \tag{62}$$

Where H_z^{mo} denotes the z-direction component of the magnetic field on the slot radiated by $K\underline{\ell}_m$ in free space. Substituting (62) into (61), we have

$$I_n^m = - 2\Delta x \int_{(n-1)\Delta z}^{(n+1)\Delta z} T_n(z)\, H_z^{mo} \, dz \tag{63}$$

For the far field, we take $K\underline{\ell}_m$ perpendicular to $\underline{r}_m$ and let $r_m \to \infty$. At

the same time we adjust K^{ℓ}_m so that it produces a unit plane wave in the vicinity of the origin. The required dipole is given by

$$\frac{1}{K^{\ell}_m} = \frac{-j\omega\varepsilon}{4\pi r_m} e^{-jkr_m} \tag{64}$$

and the plane wave field can be expressed by

$$\underline{H}^{mo} = \underline{U}_m e^{-j\underline{k}_m \cdot \underline{r}} \tag{65}$$

Here $\underline{U}_m$ is a unit vector in the direction of $\underline{H}^{mo}$, $\underline{k}_m$ is the propagation vector, and $\underline{r}$ is the radius vector to an arbitrary field point. Substituting (65) into (63), we have

$$I_n^m = -2 \iint_{slot} T_n(z)\, e^{-j\underline{k}_m \cdot \underline{r}}\, \underline{U}_m \cdot \underline{U}_z \, ds \tag{66}$$

From (59) and (64), the far-zone magnetic field is given by

$$H_m = \frac{-j\omega\varepsilon}{4\pi r_m} e^{-jkr_m} \tilde{\vec{I}}^m \vec{V} \tag{67}$$

where the elements I_n^m are given by (66), and $\vec{V}$ is given by (16).

For symmetry about the axis of the slot, we may calculate the field in x-z plane without loss of generality. Since the magnetic current has only a z direction component, there is no y direction component of the magnetic field in x-z plane. Therefore, we need only to evaluate the magnetic field on x-z plane as shown in Fig. 12. From Fig. 12, we have

$$\underline{k}_m = -\underline{U}_x k \sin\phi + \underline{U}_z k \cos\phi \tag{68}$$

$$\underline{U}_m = \underline{U}_x \cos\phi + \underline{U}_z \sin\phi \tag{69}$$

$$\underline{r} = \underline{U}_z z \tag{70}$$

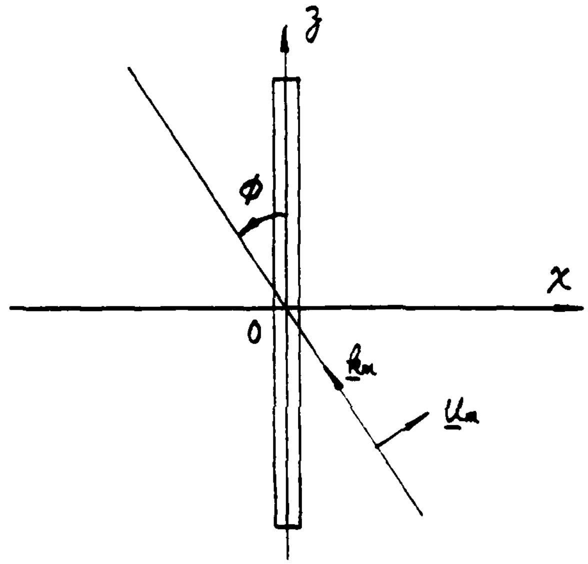


Fig. 12. Evaluation of the field in the x-z plane.

where k is the wavenumber of the excitation. Substituting (68), (69) and (70) into (66), we obtain

$$I_n^m = -2\Delta x \int_{(n-1)\Delta z}^{(n+1)\Delta z} T_n(z)\, e^{-jkz\cos\phi} \sin\phi \, dz$$

$$= \frac{-2\Delta x \sin\phi}{(k\cos\phi)^2 \Delta z} \left(2 - e^{jk\Delta z\cos\phi} - e^{-jk\Delta z\cos\phi}\right) e^{-jkn\Delta z\cos\phi}$$

$$= \frac{-8\Delta x \sin\phi \sin^2\left(\frac{k\Delta z\cos\phi}{2}\right)}{(k\cos\phi)^2\Delta z} e^{-jkn\Delta z\cos\phi}$$

$$n = 1,2,\ldots,NT \qquad (71)$$

From (71), we have

$$I_n^m = 0 \quad , \quad \phi = 0, \quad n = 1,2,\ldots,NT$$

$$I_n^m = -2\Delta x\Delta z \quad , \quad \phi = \pi/2 , \quad n = 1,2,\ldots,NT \qquad (72)$$

Substituting (67) into (56), we obtain

$$G = \frac{2\pi r_m^2}{\mathrm{Re}(P_h)} \left(\frac{\omega\varepsilon}{4\pi r_m}\right)^2 |\tilde{\vec{I}}^m \vec{V}|^2$$

$$= \frac{1}{240\lambda^2} \frac{|\tilde{\vec{I}}^m \vec{V}|^2}{\mathrm{Re}(P_h)} \qquad (73)$$

Here $\vec{I}^m$ is given by (71) and (72), $\vec{V}$ is given by (16), and $\mathrm{Re}(P_h)$ is given by (54) and (55).

Figure 13 shows the power gain patterns for ℓ/λ_o = .094, .472 under the condition where NT = 9, $m = n = 20$, $a = .02286$, $b = .1016$, $\lambda_o = 0.32$, $\varepsilon_r = 1.0$, $W = .00076$, $x_o = .01448$, and $\theta = \pi/2$.

IX. THE TRANSMISSION CROSS SECTION

The transmission cross section of the slot is defined as that area for which the incident wave contains sufficient power to produce the radiation field H_m by omnidirectional radiation over half space. For unit incident magnetic field, we have

$$\tau = \frac{2\pi r_m^2 \eta |H_m|^2}{\eta |H_o|^2}$$

$$= 2\pi r_m^2 \, |H_m|^2 \qquad (74)$$

Substituting (67) into (74), we obtain

$$\tau = \frac{\omega^2 \varepsilon^2}{8\pi} |\tilde{\vec{I}}^m \vec{V}|^2 \qquad (75)$$

where $\vec{I}^m$ is given by (71) and (72), and $\vec{V}$ is given by (16).

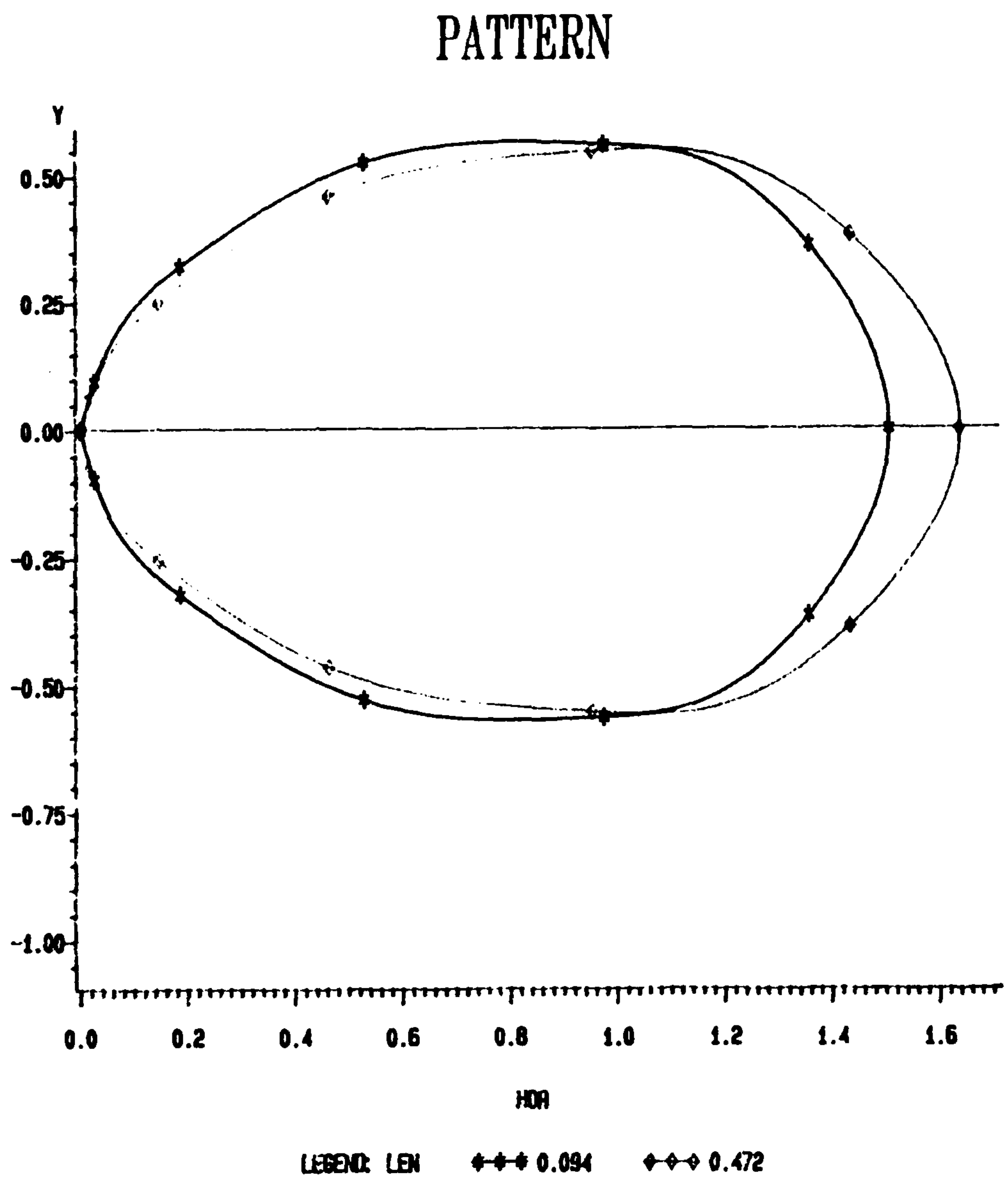


FIG.13 RERADIATION POWER PATTERN

PATTERN

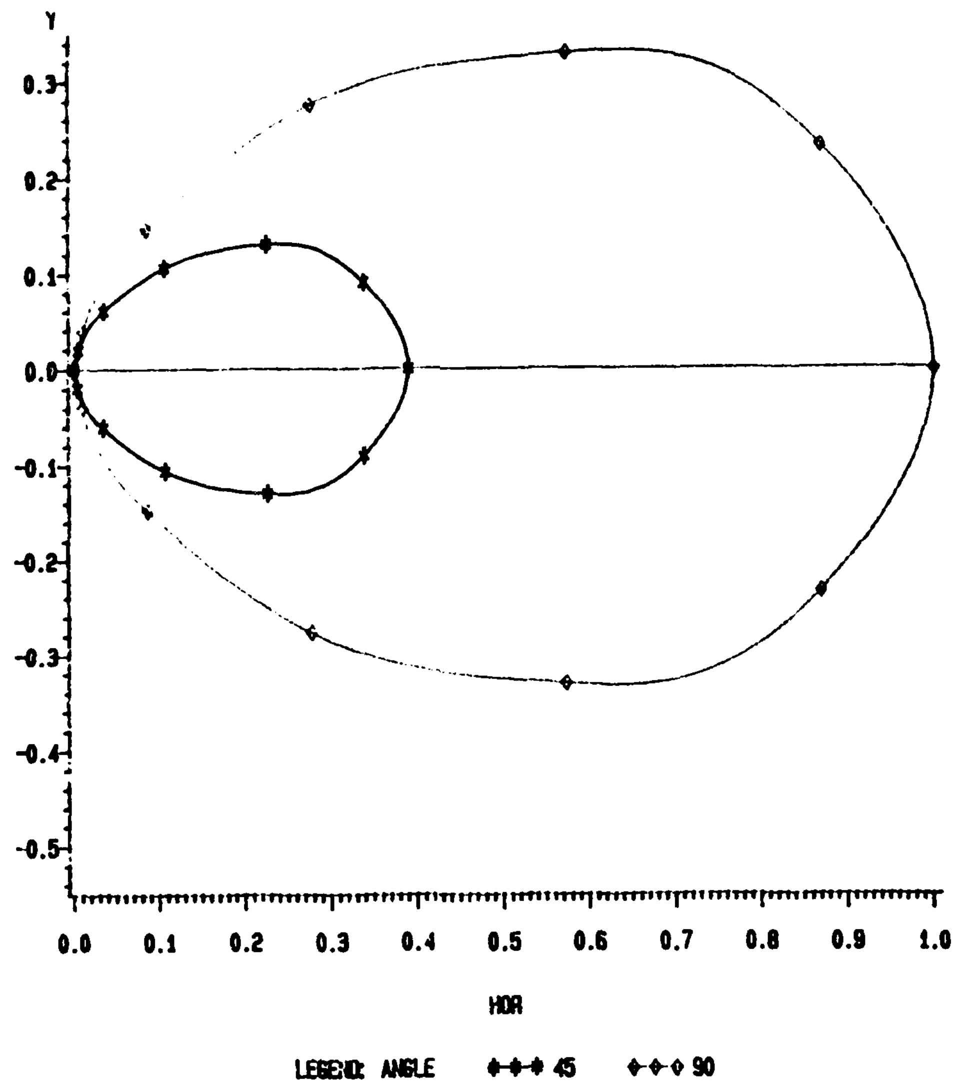


FIG.11 NORMALIZED CROSS SECTION

Figure 14 shows the normalized transmission cross sections for $\theta = \pi/4$ and $\pi/2$ under the condition NT = 9, m = n = 20, a = .02286, b = .01016, $\lambda_o = .032$, $\varepsilon_r = 1.0$, W = .00076, $x_o = .01448$, and $\ell/\lambda_o = .5$.

X. DISCUSSION

This report describes how to write the operator equation for the equivalent magnetic current over the slot of the matched waveguide by using the equivalence principle, and obtain an approximate solution to the operator equation by use of the method of moments. Then other parameters, for instance, the power penetration into the matched waveguide, the power gain pattern of the reradiation from the slot, and the resonant length of the slot, can be computed by use of the magnetic current over the slot.

We have defined three resonant lengths of the slot, i.e. ℓ_{res}, ℓ_{res1}, ℓ_{res2}. When the length of the slot is equal to ℓ_{res2}, the power penetration into the matched waveguide through the slot reaches a maximum. When the length of the slot is equal to ℓ_{res1}, the reradiation power into half space from the slot reaches a maximum. When the length of the slot is equal to ℓ_{res}, the total power produced by the slot reaches a maximum, and at the same time the components of the coefficient vector of the magnetic current have maximum amplitudes and equal phases.

A plane wave at normal incidence causes more power penetration into the waveguide than a plane wave at oblique incidence.

When the slot is near the side of the broad wall of the waveguide, orders of magnitude greater power can penetrate into the waveguide than when the same slot is near the center line of the broad wall of the waveguide.

When a slotted waveguide is in a broadband freqiency environment,

such as an electromagnetic impulse, the slotted waveguide system will possibly resonant at some frequencies of interest. At these frequencies large amounts of power will penetrate into the slotted waveguide.

These possibilities should be considered in the engineering analysis of such a system, especially for perpendicular incidence on a resonant slot near the side wall of a waveguide.

APPENDIX

I. PROGRAM DESCRIPTION

The computer program consists of a main program and three subroutines.

The main program is used to organize all the subroutines, calculate the admittance matrix inside the waveguide, and control the input and output data. Some notations in the main program are described as follows:

A and B denote the lengths of the broad and narrow side of the waveguide cross section, respectively.

XO is the displacement of the slot.

W is the width of the slot.

REP is the relative permittivity of the dielectric inside the waveguide.

PAMO represents the wavelength in free space.

M and N are indicators of the wave modes.

NT is the number of the triangle expansion functions for the slot.

RAT is the ratio of the slot length to the wavelength in free space.

T3 represents the incidence angle of the plane wave.

VM are magnitudes of the elements of the coefficient vector of the magnetic current.

VAM are values for phases of the elements of the coefficient vector.

POC is the power penetration into the matched waveguide from half space.

POH represents the reradiated power into half space from the slot.

POCW is the total power produced by the slot.

GPO represents the power gain for the reradiation wave from the slot.

GP01 is the transmission cross section of the slot

The subroutine YMAT is used for computing the admittance matrix in half space and is called by statement CALL YMAT (LX, DX, DY, DY, YXX) in the main program. Some notation in subroutine YMAT are described as follows:

LX is the number of the subsections of the slot, equal to (NT+1).

DX is the phase shift along the length of the subsection of the slot in free space.

DY is the phase shift along the width of the slot in free space.

YXX represent values of the elements for one row of the admittance matrix in half space.

The subroutine CGAUSS solves a system of linear algebraic equations and is a standard subroutine.

The subroutine DAIN is a data input subroutine and is called by the statement CALL DAIN (NP, A, B, XO, W, REP, PAMO, M, N, NT, RAT1, EK, T3, CC) in the main program.

II. PROGRAM LISTING

(1) Main Program:

```
C
C      THIS IS MAIN PROGRAM
C
       COMPLEX E,YA(10),U1,H,G3,G2,EX(10),G39,U19,G29
       COMPLEX UD,YXX(10),Y1(10,10),EX1(10),Y2(10,10)
       COMPLEX EI3,Y5(10,10),EI31,POCW,E12,E14,Y3(10,10),AS
       COMPLEX EX2(10),GP1,POH,POC,POC1(10),POH1(10)
       DIMENSION AM(10),ANN(10),GPO(10),GPO1(10)
       CALL DAIN(NP,A,B,XO,W,REP,PAMO,M,N,NT,RAT1,EK,T3,CC)
       PI=3.141593
       CC3=180/PI
       PEP=SQRT(REP)
       PAM= PAMO/PEP
       PA=PI/A
       PB=PI/B
       PX=XO*PA
       A5=2.*PI/PAM
       E=(0.0,1)*A5*PEP/(240*PI*A*B)
       A5=A5*A5
       DO 1 KP=1,NP
       RAT=RAT1+EK*KP
       PL=PAMO*RAT
       PDX=0.5*W*PA
       Z1=PL/(NT+1)
       DO 10 I=1,NT
       YA(I)=0.
10     CONTINUE
       DO 11 J3=1,N
       J2=J3-1
       IF (J3.GT.1) GO TO 12
       AN=W*W
       GO TO 14
12     AN=COS(J2*PX)/J2*SIN(J2*PDX)/PA
       AN=8*AN*AN
14     DO 15 J7=1,M
       J6=J7-1
       B5=J2*PA*J2*PA+J6*PB*J6*PB
       BA=B5-A5
       GZ2=BA*Z1*Z1
       G3=2*E*AN*Z1**3/(GZ2*GZ2)
       IF (J7.EQ.1) G3=0.5*G3
       G1=SQRT(ABS(BA))
       AA1=B5/A5
       C1=Z1*G1
       CKG=AA1/C1
```

```
      IF (BA) 30,32,32
30    SN=SIN(C1)
      CS=COS(C1)
      S1=SIN(0.5*C1)
      S2=S1*S1
      U1=-SN+2.*(0.0,1)*S2
      G2=G3*16*S2*S2*CKG
      DO 16 I=1,NT
      II=MINO(I,3)
      GO TO (18,19,20),II
18    H=2.*G3*(2.*(GZ2/3.-AA1)+CKG*(CS-3.-(0.0,1)*SN)*U1)
      GO TO 21
19    H=G3*(GZ2/3+2.*AA1+CKG*(COS(2*C1)-3*CS+4+(0.0,1)
     !*(3*SN-SIN(2*C1)))*U1)
      GO TO 21
20    A3=(I-1)*C1
      H=G2*(-SIN(A3)-(0.0,1)*COS(A3))
21    YA(I)=YA(I)+H
16    CONTINUE
      GO TO 15
32    IF (C1.GT.18) GO TO 25
      E2=EXP(-C1)
      S1=E2-1
      E22=EXP(-2*C1)
      GO TO 26
25    E2=0.0
      E22=0.0
      S1=-1
26    G2=G3*CKG
      G22=CABS(G2)
      GG2=ALOG(1.E-35)-ALOG(G22)
      DO 50 I=1,NT
      II=MINO(I,3)
      GO TO (52,53,54),II
52    H=2*G3*(2*(GZ2/3-AA1)+CKG*(E2-3)*S1)
      GO TO 55
53    H=G3*(GZ2/3+2*AA1+CKG*(E22-3*E2+4)*S1)
      GO TO 55
54    H1=-GG2/(I-2.95)
      IF (C1.GT.H1) GO TO 24
      H=G2*(EXP(-(I-3)*C1/4)-EXP(-(I+1)*C1/4))**4
      GO TO 55
24    H=0.0
55    YA(I)=YA(I)+H
50    CONTINUE
15    CONTINUE
11    CONTINUE
      CT=COS(T3)
      CT1=ABS(CT)
```

```
      IF (CT1.LE.1E-4) GO TO 96
      BAZ=2*PI/PAM*CT*Z1
      B1=W/BAZ*SIN(T3)/BAZ*Z1/(15*PI)
      SN1=SIN(.5*BAZ)
      DO 90 I=1,NT
90    EX(I)=-B1*SN1*SN1*(COS(I*BAZ)-(0.0,1)*SIN(I*BAZ))
      GO TO 91
96    EXX=-W/60*Z1/PI
      DO 98 I=1,NT
98    EX(I)=EXX
91    BK=2*PI/PAM0
      DX=BK*Z1
      DY=BK*W
      LX=NT+1
      CALL YMAT(LX,DX,DY,YXX)
      UD=(0.0,1)*Z1*W/(240*PI*PI)
      DO 80 I=1,NT
80    YXX(I)=UD*YXX(I)
      DO 58 I=1,NT
      DO 58 J=1,NT
      K=IABS(I-J)+1
      Y1(I,J)=YA(K)+YXX(K)
      Y3(I,J)=CONJG(YA(K))
58    Y5(I,J)=CONJG(YXX(K))
      DO 59 I=1,NT
59    EX1(I)=EX(I)
      DO 60 I=1,NT
      DO 60 J=1,NT
60    Y2(I,J)=Y1(I,J)
      M1=NT
      CALL CGAUSS(M1,Y2,EX1,0.1E-9,ISW)
      POC=(0.0,0.0)
      POCW=(0.0,0.0)
      POH=(0.0,0.0)
      DO 37 I=1,NT
      POC1(I)=(0.0,0.0)
      POH1(I)=(0.0,0.0)
      EI31=CONJG(EX(I))
      DO 36 J=1,NT
      EI3=CONJG(EX1(J))
      POC1(I)=POC1(I)+EI3*Y3(I,J)
36    POH1(I)=POH1(I)+EI3*Y5(I,J)
      POC=POC+EX1(I)*POC1(I)
      POCW=POCW+EX1(I)*EI31
37    POH=POH+EX1(I)*POH1(I)
      POCT=REAL(POC)
      POHT=REAL(POH)
      T2=0.0
      DO 38 I=1,7
      T2=(I-1)*PI/12
      CT2=COS(T2)
      CT3=ABS(CT2)
```

```
      IF (CT3.LE.1E-4) GO TO 40
      BAZ1=2*PI/PAM*CT2*Z1
      SN11=SIN(0.5*BAZ1)
      B11=8*W/BAZ1*SIN(T2)/BAZ1*Z1
      DO 41 J=1,NT
41    EX2(J)=-B11*SN11*SN11*(COS(J*BAZ1)-(0.0,1)*SIN(J*BAZ1))
      GO TO 42
40    EXX1=-2*W*Z1
      DO 43 J=1,NT
43    EX2(J)=EXX1
42    GP1=(0.0,0.0)
      DO 45 J=1,NT
45    GP1=GP1+EX2(J)*EX1(J)
      GP2=CABS(GP1)
      GPO(I)=GP2*GP2/(240*PAM*PAM*POHT)
38    GPO1(I)=GP2*GP2/(.288E+5*PAM*PAM)
95    DO 68 I=1,NT
      AM(I)=CABS(EX1(I))
      CC1=REAL(EX1(I))
      CC2=AIMAG(EX1(I))
      DCC=CC2/CC1
      ANN1=ATAN(DCC)
68    ANN(I)=ANN1*CC3
      WRITE (6,61) A,B,X0,W,PAMO,M,N,NT,RAT,REP,CC,T3
61    FORMAT (//2X,'A=',F7.5,2X,'B=',F7.5,2X,'X0=',F9.7,2X,
     !'W=',F9.7,2X,'PAMO=',F7.5/2X,'M=',I3,1X,'N=',I3,1X,'NT=',
     !I2,1X,'RAT=',F9.7,1X,'REP=',F5.3,1X,'CC=',F9.7,1X,'T3=',F9.7//)
      GO TO 204
205   WRITE (6,62)
62    FORMAT (1X,'YA')
      WRITE (6,63) (YA(I),I=1,NT)
      WRITE (6,65)
65    FORMAT (1X,'EX')
      WRITE (6,63) (EX(I),I=1,NT)
204   WRITE (6,69)
69    FORMAT (1X,'VM',1X,'VAN')
      WRITE (6,63) (AM(I),ANN(I),I=1,NT)
63    FORMAT (2X,6E12.4)
      WRITE (6,202)
202   FORMAT(1X,'POC',1X,'POH',1X,'POCW')
      WRITE (6,63) POC,POH,POCW
      WRITE (6,64)
64    FORMAT(1X,'GPO')
      WRITE (6,63) (GPO(I),I=1,7)
      WRITE (6,66)
66    FORMAT (1X,'GPO1')
      WRITE (6,63) (GPO1(I),I=1,7)
1     CONTINUE
      STOP
      END
```

(2) Subroutine YMAT:

```
C     LISTING OF THE SUBROUTINE YMAT
      SUBROUTINE YMAT(LX,DX,DY,YXX)
      COMPLEX U,U1,U2,U3,U4,EXX,TC(11),TX(11),YXX(10)
      DX2=DX*DX
      N=LX-1
      U=(0.,1.)
      U4=.1666667*U
      YU=.5*DY
      YUD=YU*DX
      YU2=YU*YU
      YU3=.3333333*YU
      YU4=.25*DX2+YU3*YU
      DO 16 I=1,LX
      IP=I+1
      XU=(I-.5)*DX
      XU2=XU*XU
      XL=XU-DX
      XL2=XL*XL
      R1=(I-1)*DX
      R2=R1*R1
      RU1=1.-.5*R2
      U1=RU1+R1*(1.-.1666667*R2)*U
      U2=(R1-RU1*U)*YUD
      U3=-.5-.5*R1*U
      EXX=2.*(COS(R1)-U*SIN(R1))
      R7=XL2+YU2
      R8=XU2+YU2
      R3=SQRT(R7)
      R4=SQRT(R8)
      AXU=XU*ALOG((YU+R4)/XU)
      AXL=XL*ALOG((YU+R3)/ABS(XL))
      AYU=YU*ALOG((XU+R4)/(XL+R3))
      C1=AXU-AXL+AYU
      C3=YU3*(XU*R4-XL*R3)+.1666667*(XU2*AXU-XL2*AXL+YU2*AYU)
      C4=YU3*(XU*R8-XL*R7)
      TC(IP)=(C1*U1+U2+C3*U3+C4*U4)*EXX
      AXU=XU*AXU
      AXL=XL*AXL
      X1=.5*(YU*(R4-R3)+AXU-AXL)
      X3=YU*(.8333333E-1*(R8*R4-R7*R3)+.125*(XU2*R4-XL2*R3))
     !+.125*(XU2*AXU-XL2*AXL)
      X4=R1*YUD*(R2+YU4)
      TX(IP)=(X1*U1+R1*U2+X3*U3+X4*U4)*EXX/DX
16    CONTINUE
      TC(1)=TC(3)
      TX(1)=-TX(3)
      DO 20 J=2,LX
      JM=J-1
      JP=J+1
      YXX(JM)=.5*(TC(J)-TX(JP)+(J-.5)*TC(JP)+TX(JM)-
     !(J-3.5)*TC(JM))+(TC(JP)-2.*TC(J)+TC(JM))/DX2
20    CONTINUE
      RETURN
      END
```

(3) Subroutine CGAUSS

```
C
C       THIS IS SUBROUTINE 4
C
        SUBROUTINE CGAUSS(N,A,B,EPS,ISW)
        COMPLEX A(10,10),B(10),C,T
        NM1=N-1
        DO 10 K=1,NM1
        C=(0.0,0.0)
        DO 2 I=K,N
        IF (CABS(A(I,K)).LE.CABS(C)) GO TO 2
        C=A(I,K)
        IO=I
2       CONTINUE
        IF (CABS(C).GE.EPS) GO TO 3
        ISW=0
        RETURN
3       IF (IO.EQ.K) GO TO 6
        DO 4 J=K,N
        T=A(K,J)
        A(K,J)=A(IO,J)
4       A(IO,J)=T
        T=B(K)
        B(K)=B(IO)
        B(IO)=T
6       KP1=K+1
        C=1./C
        B(K)=B(K)*C
        DO 10 J=KP1,N
        A(K,J)=A(K,J)*C
        DO 20 I=KP1,N
20      A(I,J)=A(I,J)-A(I,K)*A(K,J)
10      B(J)=B(J)-A(J,K)*B(K)
        B(N)=B(N)/A(N,N)
        DO 40 K=1,NM1
        I=N-K
        C=(0.0,0.0)
        IP1=I+1
        DO 50 J=IP1,N
50      C=C+A(I,J)*B(J)
40      B(I)=B(I)-C
        ISW=1
        RETURN
        END
```

(4) Subroutine DAIN:

```
C
C     THIS IS DATA INPUT FOR YAWA1
C
      SUBROUTINE DAIN(NP,A,B,XO,W,REP,PAMO,M,N,NT,RAT1,EK,T3,CC)
      NP=3
      CC=3*3.141593/2
      T3=3.141593/2
      A=.02286
      B=.01016
      XO=.01448
      W=.00076
      REP=1.0
      PAMO=.032
      RAT1=.4247
      EK=.0236
      M=20
      N=20
      NT=9
      RETURN
      END
```

III. PROGRAM USAGE

We write a subroutine to input the necessary data. Keeping the main program unchanged and using different data input subroutines, we can compute different parameters of the slot. The example is as follows:

```
C
C     THIS IS DATA INPUT FOR YAWA1
C
      SUBROUTINE DAIN(NP,A,B,X0,W,REP,PAMO,M,N,NT,RAT1,EK,T3,CC)
      NP=3
      CC=3*3.141593/2
      T3=3.141593/2
      A=.02286
      B=.01016
      X0=.01448
      W=.00076
      REP=1.0
      PAMO=.032
      RAT1=.42826
      EK=.02379
      M=20
      N=20
      NT=9
      RETURN
      END
```

```
 A=0.02286  B=0.01016  X0=0.0144800  W=0.0007600  PAMO=0.03200
 M= 20 N= 20 NT= 9 RAT=0.4520500 REP=1.000 CC=4.7123890 T3=1.5707960


VM VAN
   0.2075E+02  0.3640E+02  0.3419E+02  0.3629E+02  0.4432E+02  0.3621E+02
   0.5059E+02  0.3617E+02  0.5272E+02  0.3616E+02  0.5059E+02  0.3617E+02
   0.4432E+02  0.3621E+02  0.3419E+02  0.3629E+02  0.2075E+02  0.3640E+02
POC POH POCW
   0.1573E-06  0.6027E-06  0.1501E-05  0.6120E-06  0.1658E-05  0.1215E-05
GPO
   0.0000E+00  0.7351E-01  0.2970E+00  0.6609E+00  0.1101E+01  0.1478E+01
   0.1628E+01
GPO1
   0.0000E+00  0.9194E-09  0.3714E-08  0.8265E-08  0.1377E-07  0.1848E-07
   0.2036E-07
```

```
 A=0.02286  B=0.01016  X0=0.0144800  W=0.0007600  PAMO=0.03200
 M= 20 N= 20 NT= 9 RAT=0.4758400 REP=1.000 CC=4.7123890 T3=1.5707960


VM VAN
   0.2429E+02  0.2262E+00  0.4046E+02  0.7859E-01  0.5276E+02 -0.1445E-01
   0.6042E+02 -0.6759E-01  0.6304E+02 -0.8482E-01  0.6042E+02 -0.6760E-01
   0.5276E+02 -0.1448E-01  0.4046E+02  0.7856E-01  0.2429E+02  0.2262E+00
POC POH POCW
   0.2409E-06  0.1708E-06  0.2331E-05 -0.1704E-06  0.2572E-05  0.2468E-09
GPO
   0.0000E+00  0.7112E-01  0.2897E+00  0.6520E+00  0.1098E+01  0.1485E+01
   0.1641E+01
GPO1
   0.0000E+00  0.1381E-08  0.5628E-08  0.1266E-07  0.2133E-07  0.2885E-07
   0.3187E-07

 A=0.02286  B=0.01016  X0=0.0144800  W=0.0007600  PAMO=0.03200
 M= 20 N= 20 NT= 9 RAT=0.4996300 REP=1.000 CC=4.7123890 T3=1.5707960


VM VAN
   0.1974E+02 -0.3056E+02  0.3324E+02 -0.3074E+02  0.4362E+02 -0.3086E+02
   0.5014E+02 -0.3092E+02  0.5236E+02 -0.3095E+02  0.5014E+02 -0.3092E+02
   0.4362E+02 -0.3086E+02  0.3324E+02 -0.3074E+02  0.1974E+02 -0.3056E+02
POC POH POCW
   0.1770E-06 -0.3387E-06  0.1737E-05 -0.8038E-06  0.1914E-05 -0.1143E-05
GPO
   0.0000E+00  0.6870E-01  0.2824E+00  0.6429E+00  0.1095E+01  0.1493E+01
   0.1654E+01
GPO1
   0.0000E+00  0.9945E-09  0.4088E-08  0.9307E-08  0.1585E-07  0.2161E-07
   0.2395E-07
```

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